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CLASS 10

MOCK PAPER

MATHEMATICS [SA1]

Time : 3 Hrs. *MM : 90*

GENERAL INSTRUCTIONS

I. *All questions are compulsory.*
II. *The question paper consists of 34 questions divided into four sections A, B, C and D.*
III. *Section A contains 8 questions of 1 mark each, which are multiple choice type questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each, Section D contains 10 questions of 4 marks each.*
IV. *There is no overall choice in the paper. However, internal choice is provided in one question of 2 marks, three questions of 3 marks and two questions of 4 marks.*
V. *Use of calculator is not permitted.*

SECTION – A

DIRECTIONS : *Question numbers 1 to 8 carry 1 mark each. For each questions 1 to 8, four alternative choices have been provided of which only one is correct. You have to select the correct choice.*

1. Which of the following is irrational?

(a) $\frac{22}{7}$ (b) 3.141592
(c) 2.78181818....
(d) 0.123223222322223.........

2. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then
(a) $a = -7, b = -1$ (b) $a = 5, b = -1$
(c) $a = 2, b -6$ (d) $a = 0, b = -6$

3. If $\Delta ABC \sim \Delta PQR$ with $BC = 8$ cm and $QR = 12$ cm, then $\frac{area(\Delta ABC)}{area(\Delta PQR)}$ is equal to
(a) $2 : 3$ (b) $4 : 9$
(c) $8 : 27$ (d) none of these

4. $\sin(45° + \theta) - \cos(45° - \theta)$ is equal to
(a) $2\cos\theta$ (b) 0
(c) $2\sin\theta$ (d) 1

5. By Euclid's division lemma $x = qy + r, x > y$, the value of q and r for $x = 27$ and $y = 5$ are:
(a) $q = 5, r = 3$ (b) $q = 6, r = 3$
(c) $q = 5, r = 2$
(d) cannot be determined

6. The value of $\frac{\cos(90° - \theta)\cos\theta}{\tan\theta} - 1$ is
(a) $-\sin^2\theta$ (b) $-\operatorname{cosec}^2\theta$
(c) $-\cos^2\theta$ (d) $-\cot\theta$

7. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$
(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$

(c) $\dfrac{a^2+b^2}{a^2-b^2}$ (d) $\dfrac{a^2-b^2}{a^2+b^2}$

8. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is

(a) 6 (b) 7
(c) 8 (d) 10

SECTION – B

DIRECTIONS : *Question number 9 to 14 carry 2 marks each.*

9. To find the H.C.F. of 1071 and 1029, using Euclid 's division algorithm.

10. If $x=\dfrac{4}{3}$ is a root of the polynomial $f(x)=6x^3-11x^2+kx-20$ then find the value of k.

Or

Find the zeroes of the polynomial $\sqrt{3}x^2+10x+7\sqrt{3}\cdot$

11. Given $\Delta ABC \sim \Delta DEF$. If $AB = 2DE$ and area of ΔABC is 56 cm^2. find the area of ΔDEF.

12. If A, B, C are interior angles of $\Delta\ ABC$, show that: $\cos\left(\dfrac{B+C}{2}\right)=\sin\dfrac{A}{2}$

13. Draw the graphs of the pair of linear equations $x-y+2=0$ and $4x-y-4=0$. Calculate the area of the triangle formed by the lines so drawn and the x-axis.

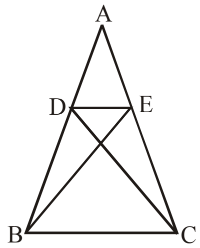
14. Construct the cumulative frequency distribution of the following distribution

Class	12.5 – 17.5	17.5 – 22.5	22.5 – 27.5	27.5 – 32.5	32.5 – 37.5
Frequency	2	22	19	14	13

SECTION – C

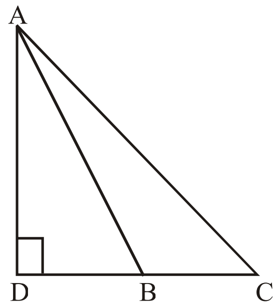
DIRECTIONS : *Question number 15 to 24 carry 3 marks each.*

15. In adjoining figure if $\Delta ABE \cong \Delta ACD$, prove that $\Delta ADE \sim \Delta ABC$



Or

In the given figure, $\angle$ ADC = 90°. Prove that AC^2 = AB^2 + BC^2 + 2BC.BD.



16. Three wheels can complete respectively 60, 36, 24 revolutions per minute. There is a red spot on each wheel that touches the ground at time zero. After how much time, all these spots will simultaneously touch the ground again?

17. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeroes as 2, – 7, – 14 respectively.

Or

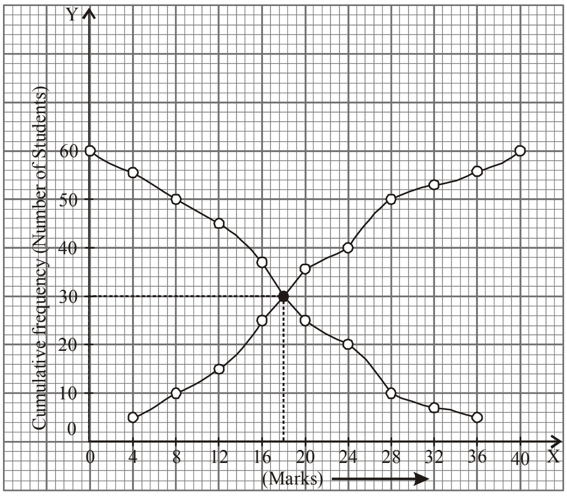
On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$ respectively. Find $g(x)$.

18. If $\sin\theta=\dfrac{3}{5}$, prove that $(\tan\theta+\sec\theta)^2=4$.

19. Prove that: $\sqrt{\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}}=\dfrac{1-\sin\theta}{\cos\theta}$

20. Solve for x and y : $\dfrac{3}{x}+\dfrac{4}{y}=1;\ \dfrac{4}{x}+\dfrac{2}{y}=\dfrac{11}{12}$

21. What is the median of the data using the graph of less than ogive and more than ogive?



22. In the given figure ABC and DBC are two triangles on the same base BC. If AD intersects BC at O.

Prove that $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$.

23. Evaluate the following:

$$\frac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ}{\tan 5^\circ \tan 30^\circ \tan 55^\circ \tan 35^\circ \tan 85^\circ}$$

24. The arithmetic mean of the following frequency distribution is 25. Determine the value of p.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	18	15	p	6

Or

Find the mean of the following data, using step-deviation method.

Class interval	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	7	8	12	10	8	5

SECTION – D

DIRECTIONS : *Question number 25 to 34 carry 4 marks each.*

25. Prove that $\sqrt{2}$ is irrational.

26. Jamila sold a table and a chair for ₹ 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got ₹ 1065. Find the cost price of each.

27. Prove that $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = \sec\theta . \text{cosec}\theta + 1$

28. A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained :

Height (in cm)	**Number of girls**
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

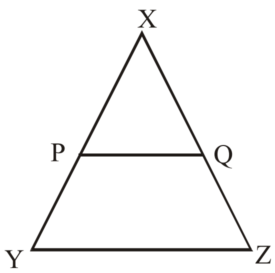
Find the median height and interpret the result.

29. A criminal start driving a stolen car from a point (8, –8) along the road given by $2x + y = 8$ and second criminal start running on a motor cycle from a point (6, 3) along the road given by $3x - 2y = 12$. If both the criminals are moving in such directions on the above mentioned road such that they meet at a point, then locate the point graphically where the police party should stay to arrest both the criminals will you advice some more police on the y-axis to arrest the criminals?

30. Through the mid-point M of the side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC at L and AD produced at E. Prove that $EL = 2BL$.

Or

In figure, $\dfrac{XP}{PY} = \dfrac{XQ}{QZ} = 3$, if the area of ΔXYZ is 32 cm^2, then find the area of the quadrilateral PYZQ.



31. If $\sec\theta = x + \frac{1}{4x}$, prove that :

$\sec\theta + \tan\theta = 2x$ or, $\frac{1}{2x}$

Or

If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$

show that $(m^2 + n^2)\cos^2\beta = n^2$

32. Prove that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

33. Find all the zero(es) of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zero(es) are $\sqrt{2}$ and $-\sqrt{2}$.

34. Find the mean marks (average marks) obtained by a student from the following

Marks	Number of students
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43
60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

HINTS & SOLUTIONS

SECTION – A

1. (d) The number $\frac{22}{7}$ is rational
3.141592 is also rational since the number is terminating decimal. 2.78181818.... is also rational since the number is non-terminating but repeating decimal. **(½ mark)**
0.123223222322223......... is irrational since the number is neither terminating nor repeating decimal. **(½ mark)**

2. (d) Sum of zeroes = 2– 3 = –1 $\therefore$ –1 $(a + 1) = -1$
$\Rightarrow a + 1 = 1 \Rightarrow a = 0$ **(½ mark)**
Product of zeroes = $-6 = b \therefore b = -6$ **(½ mark)**

3. (b) We have, $\Delta ABC \sim \Delta PQR$
$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$ **(½ mark)**
$= \frac{8^2}{12^2} = \frac{64}{144} = \frac{4}{9} = 4:9$ **(½ mark)**

4. (b) $\cos(45° - \theta) = \cos(90° - 45° - \theta)$
$= \cos(90° - (45° + \theta) = \sin(45° + \theta)$ **(½ mark)**
$\therefore$ Given expression
$= \sin(45° + \theta) - \sin(45° + \theta) = 0$ **(½ mark)**

5. (c) $x = qy + r \Rightarrow 27 = 5 \times 5 + 2 \Rightarrow q = 5, r = 2$ **(1 mark)**

6. (a) $\frac{\cos(90° - \theta)\cos\theta}{\tan\theta} - 1$
$= \frac{\sin\theta . \cos\theta}{\sin\theta / \cos\theta} - 1$ **(½ mark)**
$= \cos^2\theta - 1$
$= -(1 - \cos^2\theta)$
$= -\sin^2\theta$ **(½ mark)**

7. (d) $\tan\theta = \frac{a}{b}$
$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2 - b^2}{a^2 + b^2}$ **(1 mark)**

8. (c) Mode of the data is 8 as it is repeated maximum number of times. **(1 mark)**

SECTION – B

DIRECTIONS : *Question number 9 to 14 carry 2 marks each.*

9. Since, 1071 > 1029, we apply the Euclid's division lemma to 1071 and 1029, we get
$1071 = 1029 \times 1 + 42$ **(½ mark)**
since, remainder 42 $\neq 0$ so again applying division lemma in 1029 and 42, we get
$1029 = 42 \times 24 + 21$ again $21 \neq 0$ **(½ mark)**
Applying Euclid's Lemma again in 42 and 21, we get $42 = 21 \times 2 + 0$ **(½ mark)**
Since, remainder is zero so H.C.F. is 21. **(½ mark)**

10. $f(x) = 6x^3 - 11x^2 + kx - 20$
$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$$ **(½ mark)**
$\Rightarrow 6.\frac{64}{27} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$ **(½ mark)**
$\Rightarrow 128 - 176 + 12k - 180 = 0$ **(½ mark)**
$\Rightarrow 12k + 128 - 356 = 0 \Rightarrow 12k = 228 \Rightarrow k = 19$ **(½ mark)**

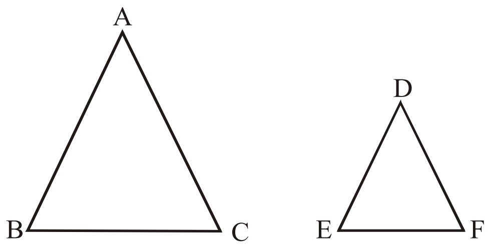
Or

We have
$f(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$
$= \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3}$
$= \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3})$
$= (\sqrt{3}x + 7)(x + \sqrt{3})$ **(½ mark)**
The zeroes of $f(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$ are given by $f(x) = 0$
$(\sqrt{3}x + 7)(x + \sqrt{3}) = 0$
$\sqrt{3}x + 7 = 0$ or $x + \sqrt{3} = 0$ **(½ mark)**
$x = -\frac{7}{\sqrt{3}}$ or $x = -\sqrt{3}$ **(½ mark)**
Hence, zeroes of $\sqrt{3}x^2 + 10x + 7\sqrt{3}$
are $n = -\sqrt{3}$ and $n = \frac{-7}{\sqrt{3}}$ **(½ mark)**

11. Given : $AB = 2DE$

$\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{area(\Delta ABC)}{area(\Delta DEF)} = \frac{AB^2}{DE^2}$$ **(½ mark)**



(½ mark)

or $$\frac{56}{area(\Delta DEF)} = \frac{4DE^2}{DE^2} = 4 \quad [\because AB = 2DE]$$

(½ mark)

area $(\Delta DEF) = \frac{56}{4} = 14$ sq.cm. **(½ mark)**

12. In $\Delta ABC, A + B + C = 180°$ **(½ mark)**

$\Rightarrow \quad B + C = 180° - A$

$\Rightarrow \quad \frac{B+C}{2} = \frac{180° - A}{2}$ **(½ mark)**

$\Rightarrow \quad \frac{B+C}{2} = 90° - \frac{A}{2}$ **(½ mark)**

$\Rightarrow \quad \cos\frac{B+C}{2} = \cos\left(90° - \frac{A}{2}\right) = \sin\frac{A}{2}.$

(½ mark)

13. For drawing the graphs of the given equations, we find two solutions of each of the equations, which are given in Table

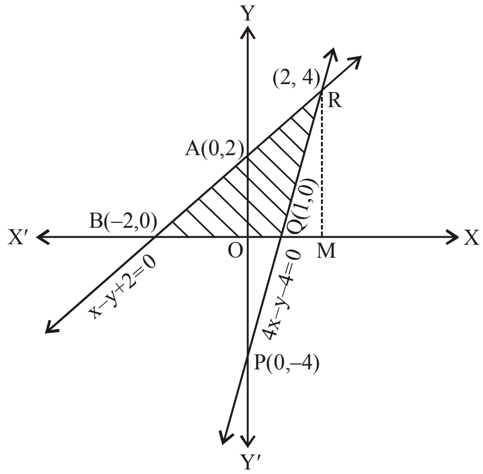
Table

***x* :**	0	–2
***y* = x + 2**	2	0

***x* :**	0	1
***y* = 4x – 4**	–4	0

(½ mark)

Plot the points A (0, 2) , B (–2, 0), P (0, – 4) and Q (1, 0) on the graph paper, and join the points to form the lines AB and PQ as shown in figure



(1 mark)

We observe that there is a point R (2, 4) common to both the lines AB and PQ. The triangle formed by these lines and the x-axis is BQR. The vertices of this triangle are B (–2, 0), Q (1, 0) and R (2, 4). We know that;

Area of triangle = $\frac{1}{2}$ × Base × Altitude

Here, Base = $BQ = BO + OQ = 2 + 1 = 3$ units.

Altitude = RM = Ordinate of R = 4 units.

So, area of $\Delta BQR = \frac{1}{2} \times 3 \times 4 = 6$ sq. units

(½ mark)

14. The required cumulative frequency distribution of the given distribution is given below:

Class	Frequency	Cumulative frequency
12.5 – 17.5	2	2
17.5 – 22.5	22	24
22.5 – 27.5	19	43
27.5 – 32.5	14	57
32.5 – 37.5	13	70

(2 marks)

SECTION – C

DIRECTIONS : *Question number 15 to 24 carry 3 marks each.*

15. We are given that $ABE \cong \Delta ACD$

Therefore, $AB = AC$ (CPCT) ... (1)

and $AE = AD$ (CPCT) ... (2)

(½ mark)

$\therefore \quad \dfrac{AB}{AD} = \dfrac{AC}{AE}$ [From (1) and (2)]

(½ mark)

i.e., $\dfrac{AB}{AC} = \dfrac{AD}{AE}$ ...(3)

(½ mark)

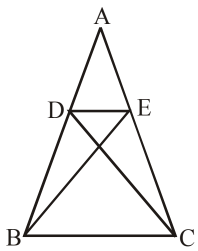
Now in ΔADE, $\angle A$ (i.e., $\angle DAE$) is included between sides AD and AE and in ΔABC. $\angle A$ (i.e., $\angle BAC$) is included between sides AB and AC and $\angle DAE = \angle BAC$ (Common angles)

(½ mark)

Further $\dfrac{AB}{AC} = \dfrac{AD}{AE}$ [From (3)]

(½ mark)

$\therefore \quad \Delta ADE \sim \Delta ABC$ (SAS similarity)



(½ mark)

Or

In ΔADC, $\angle D = 90°$

We have,

$AC^2 = AD^2 + DC^2$ ...[By Pythagoras theorem]

$AD^2 = AC^2 - DC^2$ ...(1) **(½ mark)**

In ΔADB, $\angle D = 90°$

We have,

$AB^2 = AD^2 + DB^2$ [By Pythagoras theorem]

$AB^2 = (AC^2 - DC^2) + DB^2$ [Using 1] **(½ mark)**

$AC^2 = AB^2 + (DC^2 - DB^2)$

$= AB^2 + (DC - DB)(DC + DB)$ **(½ mark)**

$\left[\because a^2 - b^2 = (a-b)(a+b)\right]$

$= AB^2 + [DB + (DB + BC)]$

$[(DC - (DC - BC)]$

(½ mark)

$AC^2 = AB^2 + (2DB + BC)\,BC$

$AC^2 = AB^2 + 2DB.BC) + BC^2$ **(½ mark)**

$AC^2 = AB^2 + BC^2 + 2BC.BD$ $[\because BD = DB]$

Hence proved. **(½ mark)**

16. 1st wheel makes 1 revolutions per sec

(½ mark)

2nd wheel makes $\dfrac{6}{10}$ revolutions per sec

(½ mark)

3rd wheel makes $\dfrac{4}{10}$ revolutions per sec

(½ mark)

In other words 1st, 2nd and 3rd wheel take 1, $\dfrac{5}{3}$ and $\dfrac{5}{2}$ seconds respectively to complete one revolution. **(½ mark)**

L.C.M of 1, $\dfrac{5}{3}$ and $\dfrac{5}{2} = \dfrac{\text{L.C.M. of 1, 5, 5}}{\text{H.C.F. of 1, 3, 2}} = 5$

(½ mark)

Hence, after every 5 seconds the red spot of all the three wheels touch the ground. **(½ mark)**

17. Let the cubic polynomial be, $ax^3 + bx^2 + cx + d$ and let is zeroes be α, β, γ **(½ mark)**

Given, $\alpha+\beta+\gamma = 2, \alpha\beta+\beta\gamma+\gamma\alpha = -7, \alpha\beta\gamma = -14$

(½ mark)

$\therefore \quad ax^3 + bx^2 + cx + d = K(x-\alpha)(x-\beta)(x-\gamma)$

$= K[x^3 - (\alpha+\beta+\gamma)x^2$

$+ (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma]$

$= K[x^3 - 2x^2 - 7x + 14]$ **(1 mark)**

For different real values of K, we can have different cubics all having the sum, sum of the product of its zeroes taken two at a time and product of its zeroes as 2, – 7, –14 respectively.

(1 mark)

Or

Here $p(x) = x^3 - 3x^2 + x + 2$, $g(x) = ?$

$q(x) = (x-2)$ and $r(x) = -2x + 4$ **(½ mark)**

By Division Algorithm, we have,

$p(x) = g(x).\,q(x) + r(x) \Rightarrow p(x) - r(x) = g(x).\,q(x)$

(½ mark)

$\Rightarrow \quad g(x) = \dfrac{p(x) - r(x)}{q(x)}$ **(½ mark)**

$= \dfrac{(x^3 - 3x^2 + x + 2) - (-2x + 4)}{(x-2)}$ **(½ mark)**

$\Rightarrow \quad g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x - 2}$

$\Rightarrow \quad g(x) = x^2 - x + 1$ **(½ mark)**

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & \underset{-}{x^3} \underset{+}{-\,2x^2} \\ \hline & -x^2 + 3x - 2 \\ & \underset{+}{-x^2} \underset{-}{+\,2x} \\ \hline & x - 2 \\ & \underset{-}{x} \underset{+}{-\,2} \\ \hline & \text{zero} \end{array}$$

(½ mark)

18. $(\tan\theta + \sec\theta)^2 = \left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)^2 = \left(\dfrac{1+\sin\theta}{\cos\theta}\right)^2$

(1 mark)

$$= \frac{(1+\sin\theta)^2}{\cos^2\theta} = \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$$ **(1 mark)**

$$= \frac{\left(1+\frac{3}{5}\right)^2}{1-\left(\frac{3}{5}\right)^2} = \frac{\left(\frac{64}{25}\right)}{\left(\frac{16}{25}\right)} = \frac{64}{16} = 4.$$ **(1 mark)**

19. We have

$$\text{LHS} = \sqrt{\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}}$$

$$= \sqrt{\frac{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}}$$ **(½ mark)**

$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$ **(½ mark)**

$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$$ **(½ mark)**

$$= \frac{1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$ **(½ mark)**

$$= \frac{1-\sin\theta}{\sqrt{\cos^2\theta}}$$ **(½ mark)**

$$= \frac{1-\sin\theta}{\cos\theta}$$ **(½ mark)**

$= \text{R.H.S.}$

20. $\frac{3}{x} + \frac{4}{y} = 1$ ...(1) **(½ mark)**

$\frac{4}{x} + \frac{2}{y} = \frac{11}{12}$ ...(2) **(½ mark)**

Multiplying (2) by 2 $\Rightarrow \frac{8}{x} + \frac{4}{y} = \frac{22}{12}$ ...(3) **(½ mark)**

Subtracting (1) and (3) $\Rightarrow \frac{5}{x} = \frac{10}{12}$

$\therefore\ x = \frac{5\times 12}{10} = 6$ **(½ mark)**

Substituting $x = 6$ in (1)

$\Rightarrow\ \frac{3}{6} + \frac{4}{y} = 1$

$\Rightarrow\ \frac{4}{y} = 1 - \frac{1}{2} = \frac{1}{2}$ **(½ mark)**

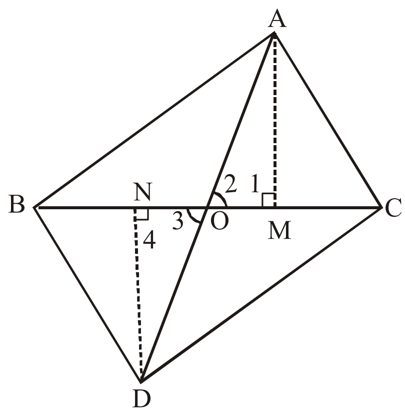
$\therefore\ y = 8$ Hence, $x = 6$ and $y = 8$ **(½ mark)**

21. Less than ogive and more than ogive intersect each other at the point $(x = 18, y = 30)$. **(1 mark)**
Also, $n = 60$

$\Rightarrow\ \frac{n}{2} = 30$ **(½ mark)**

$\Rightarrow$ The middle most value is 30th **(½ mark)**
Thus, corresponding to $y = 30$, we have $x = 18$.
Hence, median = 18 marks. **(1 mark)**

22. Draw $AM \perp BC$, $DN \perp BC$ In ΔAMO and ΔDNO,



(1 mark)

$\angle 1 = \angle 4$ ......(each 90°)
$\angle 2 = \angle 3$ ......(vetically opposite $\angle s$)
$\Delta AMO \sim \Delta DNO$ .....(By A A rule of similarity) **(½ mark)**

$$\frac{AO}{DO} = \frac{AM}{DN} \quad \text{......(i)}$$ **(½ mark)**

Now,

$$\frac{area(\Delta ABC)}{area(\Delta DBC)} = \frac{(1/2)(BC)(AM)}{(1/2)(BC)(DN)} = \frac{AM}{DN} = \frac{AO}{DO}$$ ......from (i)

Hence, $\frac{area(\Delta ABC)}{area(\Delta DBC)} = \frac{AO}{DO}$ **(1 mark)**

23. Given expression

$$= \frac{\sin 15^\circ \cos(90^\circ - 15^\circ) + \cos 15^\circ \sin(90 - 15^\circ)}{\tan 5^\circ \frac{1}{\sqrt{3}} \tan(35^\circ)\tan(90^\circ - 35^\circ)\tan(90^\circ - 5^\circ)}$$ **(1 mark)**

$$= \frac{\sin 15^\circ \sin 15^\circ + \cos 15^\circ \cos 15^\circ}{(\tan 5^\circ \cot 5^\circ)(\tan 35^\circ \cot 35^\circ).\frac{1}{\sqrt{3}}}$$ **(1 mark)**

$$= \frac{\sin^2 15^\circ + \cos^2 15^\circ}{(1)(1).\frac{1}{\sqrt{3}}} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$ **(1 mark)**

24. We have,

Class interval	Frequency f_i	Midvalue x_i	$(f_i \times x_i)$
0 – 10	5	5	25
10 – 20	18	15	270
20 – 30	15	25	375
30 – 40	p	35	35p
40 – 50	6	45	270
	$\Sigma f_i = (44 + p)$		$\Sigma (f_i \times x_i) = (940 + 35p)$

(½ mark)

$\therefore$ Mean, $\bar{x} = \dfrac{\sum (f_i \times x_i)}{\sum f_i}$ **(½ mark)**

$\Rightarrow \dfrac{(940 + 35p)}{(44 + p)} = 25$ **(½ mark)**

$\Rightarrow (940 + 35p) = 25\,(44 + p)$

$\Rightarrow (35p - 25p) = (1100 - 940)$ **(½ mark)**

$\Rightarrow 10p = 160 \Rightarrow p = 16$

Hence, $p = 16$ **(½ mark)**

Since the graph of the given polynomial intersects X-axis at $x = -2$, 0 and 2. Therefore, zeroes of the given cubic polynomial are -2, 0 and 2. **(½ mark)**

Or

x_i	f_i	$u_i = \dfrac{x_i - A}{h}$	$f_i u_i$
10	7	– 2	– 14
30	8	– 1	– 8
50	12	0	0
70	10	1	10
90	8	2	16
110	5	3	15
	$\Sigma f_i = 50$		$\Sigma f_i u_i = 19$

(1 mark)

Here, A = 50, h = 20

$\Sigma f_i = 50$, $\Sigma f_i u_i = 19$ **(½ mark)**

Mean $(\bar{x}) = A + \left(\dfrac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$ **(½ mark)**

$\bar{x} = 50 + \dfrac{19}{50} \times 20$ **(½ mark)**

$\bar{x} = 57.6$ **(½ mark)**

SECTION – D

DIRECTIONS : *Question number 25 to 34 carry 4 marks each.*

25. Let us assume, to the contrary, that $\sqrt{2}$ is rational. **(½ mark)**

So, we can find integers r and s $(\neq 0)$ such that $\sqrt{2} = \dfrac{r}{s}$. **(½ mark)**

Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get $\sqrt{2} = \dfrac{a}{b}$, where a and b are co-prime.

So, $b\sqrt{2} = a$ **(½ mark)**

Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2. Now, by it follows that 2 divides a. **(½ mark)**

If p be a prime number and divides a^2, then p also divides a if a is a positive integer.

So, we can write $a = 2c$ for some integer c.

Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. **(½ mark)**

This means that 2 divides b^2, and so 2 divides b

Therefore, a and b have at least 2 as a common factor. **(½ mark)**

But this contradicts the fact that a and b have no common factors other than 1. **(½ mark)**

This contradiction, has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. **(½ mark)**

So, we conclude that $\sqrt{2}$ is irrational.

26. Let the cost price of the table be ₹ x and the cost price of the chair be ₹ y.

The selling price of the table, when it is sold at a profit of 10%

$= ₹\left(x + \dfrac{10}{100}x\right) = ₹\,\dfrac{110}{100}x$ **(½ mark)**

The selling price of the chair when it is sold at a profit of 25%

$= ₹\left(y + \dfrac{25}{100}y\right) = ₹\,\dfrac{125}{100}y$ **(½ mark)**

So, $\dfrac{110}{100}x + \dfrac{125}{100}y = 1050$ ...(1) **(½ mark)**

When the table is sold at a profit of 25 %, its selling price $= ₹\left(x + \dfrac{25}{100}x\right) = ₹\,\dfrac{125}{100}x$

When the chair is sold at a profit of 10%, its selling price $= ₹\left(y + \dfrac{10}{100}y\right) = ₹\,\dfrac{110}{100}y$ **(½ mark)**

So, $\frac{125}{100}x+\frac{110}{100}y=1065$ ...(2) **(½ mark)**

From equations (1) and (2), we get

$110x+125y=105000$

and $125x+110y=106500$ **(½ mark)**

On adding and subtracting these equations, we get

$235x+235y=211500$

and $15x-15y=1500$

i.e. $x+y=900$ ...(3)

and $x-y=100$ ...(4)

Solving Equations (3) and (4), we get **(½ mark)**

$x=500, y=400$

So, the cost price of the table is ₹ 500 and the cost price of the chair is ₹ 400. **(½ mark)**

27. LHS $=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}-\frac{\cot\theta}{\tan\theta-1}$

(½ mark)

$=\frac{\tan^2\theta}{\tan\theta-1}-\frac{\cot\theta}{\tan\theta-1}=\frac{\tan^2\theta-\cot\theta}{\tan\theta-1}$ **(½ mark)**

$=\frac{\tan^2\theta-\frac{1}{\tan\theta}}{\tan\theta-1}=\frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$ **(½ mark)**

$=\frac{(\tan\theta-1)(\tan^2\theta+1+\tan\theta)}{\tan\theta(\tan\theta-1)}=\frac{\tan^2\theta+1+\tan\theta}{\tan\theta}$

(1 mark)

$=\frac{(1+\tan^2\theta)+\tan\theta}{\tan\theta}=\frac{\sec^2\theta+\tan\theta}{\tan\theta}$ **(½ mark)**

$=\frac{\sec^2\theta}{\tan\theta}+\frac{\tan\theta}{\tan\theta}=\frac{\sec^2\theta.\cos\theta}{\sin\theta}+1$ **(½ mark)**

$=\sec\theta.\operatorname{cosec}\theta+1=$ R.H.S. **(½ mark)**

28. To calculate the median height, we need to find the class intervals and their corresponding frequencies. **(½ mark)**

Frequency distribution table with the given cumulative frequencies is given below:

Class intervals	**Frequency**	**Cumulative frequency**
135 – 140	4	4
140 – 145	7	11
145 – 150	18	29
150 – 155	11	40
155 – 160	6	46
160 – 165	5	51

(½ mark)

Here, n = 51. So, $\frac{n}{2}=\frac{51}{2}=25.5$. This observation lies in the class 145 - 150. Then,

l (the lower limit) = 145, **(½ mark)**

c.f (the cumulative frequency of the class preceding

140 – 145) = 11, **(½ mark)**

f (the frequency of the median class 145 – 150) = 18,

h (the class size) = 5. **(½ mark)**

Using the formula, Median

$=l+\left(\frac{\frac{n}{2}-c.f}{f}\right)\times h=149.03$ **(½ mark)**

This means that the height of 50% girls is less than the height 149.03 cm and 50% are taller than the height 149.03 cm. **(1 mark)**

29. We have,

$2x+y=8$

$y=8-2x$

x	1	2	3	4	8
y	6	4	6	0	–8

(½ mark)

and

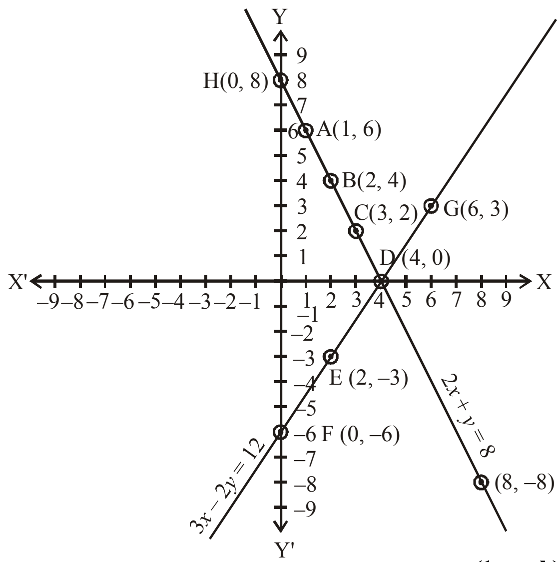
$3x-2y=12$

$2y=3x-12$

$y=\frac{3x-12}{2}$

x	4	2	0	6
y	0	–3	–6	3

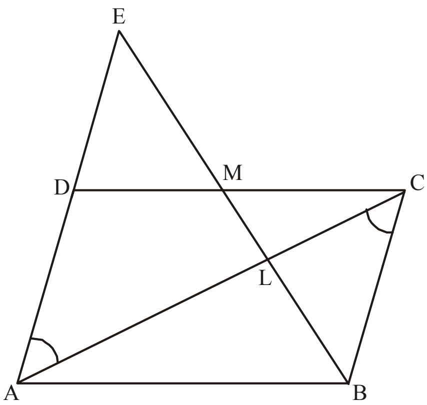
(½ mark)



(1 mark)

From the graph of the two equations, the two lines intersect each other at the point D (4, 0). Hence both criminals will pass through the point D(4, 0) on the x-axis. Hence police party should stay at D(0, 4) to arrest the criminals. **(½ mark)**
Further the criminals in stolen car will pass through H(0, 8) and on the motor cycle through the point F(0, –6). **(½ mark)**
Additional police should be posted at H and F on y-axis to arrest the criminals if they escape luckily at D. **(1 mark)**

30. In Δ's BMC and EMD, we have



(½ mark)

$\angle BMC = \angle EMD$ [Vertically opposite angles]
$MC = MD$ [$\because$ M is the mid-point of CD]
$\angle$ MBC = $\angle$ DEM [Alternate angles]
(1 mark)

So, by AAS-congruence criterion, we have

$\Delta BMC \cong \Delta EMD$
$BC = ED$
[$\because$ Corresponding parts of congruent triangles are equal]

In Δ's AEL and CBL, we have
$\angle ALE = \angle CLB$ [Vertically opposite angles]
$\angle EAL = \angle BCL$ [Alternate angles]
(1 mark)

So, by AA-criterion of similarity, we have
$\Delta AEL \sim \Delta CBL$

$$\Rightarrow \frac{AE}{BC} = \frac{EL}{BL} = \frac{AL}{CL}$$ **(½ mark)**

Taking first two terms, we get

$$\frac{EL}{BL} = \frac{AE}{BC} = \frac{AD+DE}{BC} = \frac{BC+DE}{BC} = \frac{2BC}{BC} = 2$$
(½ mark)

$\Rightarrow EL = 2BL.$ **(½ mark)**

OR

We have,

$$\frac{XP}{PY} = \frac{XQ}{QZ}$$

$\therefore$ PQ || YZ [By converse of BPT]
$\therefore$ $\angle XPQ = \angle XYZ$ [Corresponding angles]
$\angle X = \angle X$ [Common] **(1 mark)**
$\therefore$ $\Delta XPQ \sim \Delta XYZ$ [By AA Similarity]

$$\Rightarrow \frac{\text{area}(\Delta XPQ)}{\text{area}(\Delta XYZ)} = \frac{(XP)^2}{(XY)^2} = \frac{(XQ)^2}{(XZ)^2} = \frac{(PQ)^2}{(YZ)^2} \quad ...(1)$$
(½ mark)

[$\because$ The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides]

Now, $\frac{XP}{PY} = \frac{XQ}{QZ} = \frac{3}{1}$ (given)

$$\Rightarrow \frac{PY}{XP} = \frac{QZ}{XQ} = \frac{1}{3}$$

$$\Rightarrow \frac{PY}{XP} + 1 = \frac{QZ}{XQ} + 1 = \frac{1}{3} + 1$$ **(½ mark)**

$$\Rightarrow \frac{PY + XP}{XP} = \frac{QZ + XQ}{XQ} = \frac{4}{3}$$

$$\Rightarrow \frac{XY}{XP} = \frac{XZ}{XQ} = \frac{4}{3}$$

$$\Rightarrow \frac{XY}{XP} = \frac{XQ}{XZ} = \frac{3}{4}$$ **(½ mark)**

From (1) and (2), we have

$$\frac{\text{area}(\Delta XPQ)}{\text{area}(\Delta XYZ)} = \frac{(XP)^2}{(XY)^2} = \frac{(3)^2}{(4)^2} = \frac{9}{16}$$ **(½ mark)**

$$\Rightarrow \text{area}(\Delta XPQ) = \frac{9}{16} \times \text{area}(\Delta XYZ)$$

$= \frac{9}{16} \times 32$ [$\because$ area Δ XYZ = 32 cm^2]
$= 18$ cm^2 **(½ mark)**

$\Rightarrow$ area (quad. PYZQ)
= area (ΔXYZ) – area (ΔXPQ)
= 32 – 18
= 14 cm^2 **(½ mark)**

31. We have,

$$\sec\theta = x + \frac{1}{4x}$$

$$\therefore \tan^2\theta = \sec^2\theta - 1$$

$$\Rightarrow \tan^2\theta = \left(x + \frac{1}{4x}\right)^2 - 1$$

(½ mark)

$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$ **(½ mark)**

$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$

$\Rightarrow \quad \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$ **(½ mark)**

$\Rightarrow \quad \tan\theta = \pm\left(x - \frac{1}{4x}\right)$ **(½ mark)**

$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right)$ or, $\tan\theta = -\left(x - \frac{1}{4x}\right)$

(½ mark)

When $\tan\theta = \left(x - \frac{1}{4x}\right)$

We have

$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left(x - \frac{1}{4x}\right) = 2x$

(½ mark)

When, $\tan\theta = -\left(x - \frac{1}{4x}\right)$, we have

$\Rightarrow \quad \sec\theta + \tan\theta$

$= \left(x + \frac{1}{4x}\right) - \left(x - \frac{1}{4x}\right) = \frac{2}{4x} = \frac{1}{2x}$ **(½ mark)**

Hence, $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$ **(½ mark)**

Or

We have,

LHS $= (m^2 + n^2)\cos^2\beta$

$\Rightarrow \quad \text{LHS} = \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$ **(½ mark)**

$\left[\because m = \frac{\cos\alpha}{\cos\beta} \text{ and } n = \frac{\cos\alpha}{\sin\beta}\right]$

$\Rightarrow \quad \text{LHS} = \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

(½ mark)

$\Rightarrow \quad \text{LHS} = \cos^2\alpha\left(\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

(½ mark)

$\Rightarrow \quad \text{LHS} = \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

(½ mark)

$\Rightarrow \quad \text{LHS} = \frac{\cos^2\alpha}{\sin^2\beta}$ **(½ mark)**

$= \left(\frac{\cos\alpha}{\sin\beta}\right)^2$ **(½ mark)**

$= n^2 = \text{RHS}$ **(½ mark)**

Hence $(m^2 + n^2)\cos^2\beta = n^2$ **(½ mark)**

32. **Given :** A right-angled triangle ABC in which $\angle B = 90°$

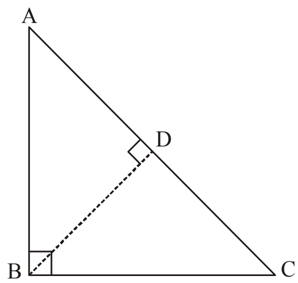
To Prove : $AC^2 = AB^2 + BC^2$

Construction : From B draw $BD \perp AC$. **(½ mark)**

Proof : In triangles ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90°]

and, $\angle A = \angle A$ [Common]



$\Rightarrow \quad \Delta ADB \sim \Delta ABC$ [AA-similarity criterion]

(½ mark)

$\Rightarrow \quad \frac{AD}{AB} = \frac{AB}{AC}$

[$\because$ In similar triangles corresponding sides are proportional] **(½ mark)**

$\Rightarrow \quad AB^2 = AD \times AC$ ...(1)

(½ mark)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90°]

and $\angle C = \angle C$ [Common]

$\Rightarrow \quad \Delta BDC \sim \Delta ABC$ [AA-similarity criterion]

(½ mark)

$\Rightarrow \quad \frac{DC}{BC} = \frac{BC}{AC}$

[$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow \quad BC^2 = AC \times DC$ ...(2) **(½ mark)**

Adding equations (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$= AC(AD + DC) = AC \times AC = AC^2$

Hence, $AC^2 = AB^2 + BC^2$ Proved. **(½ mark)**

33. Two zeroes are $\sqrt{2}$ and $-\sqrt{2}$

Now, $(x - \sqrt{2}), (x + \sqrt{2}) = x^2 - 2$, is a factor of the given polynomial. **(½ mark)**

Now, we divide the given polynomial by $x^2 - 2$

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 1 \\
\hline
x^2 - 2 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
 & 2x^4 \qquad\quad - 4x^2 \\
 & - \qquad\qquad + \\
\hline
 & -3x^3 + x^2 + 6x - 2 \\
 & -3x^3 \qquad + 6x \\
 & + \qquad\quad - \\
\hline
 & x^2 \qquad - 2 \\
 & x^2 \qquad - 2 \\
 & - \qquad\ + \\
\hline
 & 0 \\
\hline
\end{array}
$$

(1 mark)

(First term of quotient is $\frac{2x^4}{x^2} = 2x^2$)

(Second term of quotient is $\frac{-3x^3}{x^2} = -3x$)

(Third term of quotient is $\frac{x^2}{x^2} = 1$) **(½ mark)**

So, $2x^4 - 3x^3 - 3x^2 + 6x - 2$
$= (x^2 - 2)\ (2x^2 - 3x + 1)$.

Now, by splitting $-3x$, we factorise $2x^2 - 3x + 1$ as $(2x - 1)\ (x - 1)$. **(1 mark)**

So, the remaining two zero(es) are given by $x = \frac{1}{2}$ and $x = 1$. Therefore, the zero(es) of the given polynomial are $\sqrt{2}, -\sqrt{2}, \frac{1}{2}$ and 1. **(1 mark)**

34. By selecting classes for the given data as 0 – 10, 10 – 20, 20 – 30, 30 – 40, 40 – 50, 50 – 60, 60 – 70, 70 – 80, 80 – 90, 90 – 100, we write the frequency distribution as below: **(½ mark)**

Marks	Number of students f_i
0 – 10	80 – 77 = 3
10 – 20	77 – 72 = 5
20 – 30	72 – 65 = 7
30 – 40	65 – 55 = 10
40 – 50	55 – 43 = 12
50 – 60	43 – 28 = 15
60 – 70	28 – 16 = 12
70 – 80	16 – 10 = 6
80 – 90	10 – 8 = 2
90 – 100	8 – 0 = 8
Total	**n = 80**

(½ mark)

Note that in the given table there are 80 students who are getting marks 0 and above but 77 students are getting marks 10 and above. Therefore, the number of students getting marks 0 – 10, i.e., 0 and above but less than $10 = 80 - 77 = 3$.

Similarly, there are 77 students getting marks 10 and above but 72 students are getting marks 20 and above. Therefore, the number of students getting marks 10 – 20 are equal to $77 - 72 = 5$ and so on. **(½ mark)**

Let us apply step-deviation method:

$x_1 = 5, x_2 = 15, x_3 = 25, x_4 = 35, x_5 = 45,$
$x_6 = 55, x_7 = 65, x_8 = 75, x_9 = 85, x_{10} = 95.$ **(½ mark)**

We select $a = 55$ and $h = 10$.

Writting , $u_i = \frac{x_i - 55}{10}$, we make the following table: **(½ mark)**

Class marks	Frequency (Number of students) f_i	Class mark x_i	$u_i = \frac{x_i - 55}{10}$	$f_i \times u_i$
0 – 10	3	5	–5	–15
10 – 20	5	15	–4	–20
20 – 30	7	25	–3	–21
30 – 40	10	35	–2	–20
40 – 50	12	45	–1	–12
50 – 60	15	55	0	0
60 – 70	12	65	1	12
70 – 80	6	75	2	12
80 – 90	2	85	3	6
90 – 100	8	95	4	32
Total	**n = 80**			–26

(½ mark)

From the table, we have

$n = \sum f_i = 80$ and $\sum f_i u_i = -26$

By step-deviation method, the average marks of a student are given by

$$\bar{x} = a + h \times \frac{1}{n} \times \sum f_i u_i = 55 + 10 \times \frac{1}{80} \times (-26)$$

(½ mark)

$$= 55 - \frac{26}{8} = 55 - 3.25 = \mathbf{51.75}$$ **(½ mark)**